Derivace funkce

Příklad 1. Zderivujte bez úpravy (takže ten, kdo zderivovaný výraz upraví, to má špatně):

$$f(x) = \frac{5x^2}{\sqrt[5]{x}} + 30\sqrt[15]{x} + \frac{6}{\sqrt[3]{x}},$$

$$f(x) = \frac{x^3 + 2x^2 - 1}{x^4 + 2},$$

$$f(x) = \sqrt[4]{1 + \sqrt[3]{1 + \sqrt{x}}},$$

$$f(x) = \sin^2(3x + 5),$$

$$f(x) = \operatorname{tg}(x^2 + 1),$$

$$f(x) = \sin^3(\cos^2(\operatorname{tg} x)),$$

$$f(x) = x \sin^2(x^3) \ln(x^2),$$

$$f(x) = x^x,$$

$$f(x) = 3^{\ln x^2},$$

$$f(x) = 10^{x \operatorname{tg} x},$$

$$f(x) = \cos^2(x^3 - 2\sin x),$$

$$f(x) = \left(\frac{x+3}{\sqrt{5}}\right)^2 + \mathrm{e}^{-5},$$

$$f(x) = 2^{\frac{x}{\ln x}} + \mathrm{e}^{\sqrt{\ln x}},$$

$$f(x) = \mathrm{e}^{-\frac{\sqrt{5}}{2}} + (\sqrt{3})^x,$$

$$f(x) = (\sin x)^{\frac{x}{\ln x}},$$

$$f(x) = \sqrt[3]{x^2\sqrt{x^4\sqrt{x^3}}},$$

$$f(x) = (5x^4 - 3x^3 + 2x - 11)^6,$$

$$f(x) = \cos(5x + 3),$$

$$f(x) = \sin^3(x^5),$$

$$f(x) = \sin(\sin(\sin x)),$$

$$f(x) = \frac{\cos^2 x}{\cos x^2},$$

$$f(x) = \sqrt{x \sin x \sqrt{1 - x^2}},$$

$$f(x) = \log_2^2 x,$$

$$f(x) = \ln \operatorname{arctg} \frac{1}{\sqrt{x}},$$

$$f(x) = \frac{2}{\sqrt{\operatorname{tg} x}},$$

$$f(x) = \ln(-x) + \mathrm{e}^{-x^2 + \mathrm{e}^{-x}},$$

$$f(x) = \sqrt[3]{\ln \operatorname{tg} \frac{x+3}{4}},$$

$$f(x) = 5^x \cdot x^5 \cdot \sin x,$$

$$f(x) = (x^x)^x,$$

$$f(x) = \sin(\sqrt{2})^3.$$

Příklad 2. Zderivujte a upravte:

$$f(x) = x\arcsin\sqrt{\frac{x}{1+x}} + \operatorname{arctg}\sqrt{x} - \sqrt{x}, \qquad f'(x) = \arcsin\sqrt{\frac{x}{1+x}},$$

$$f(x) = \sqrt{x\sqrt{x\sqrt{x}}}, \qquad f'(x) = \frac{7}{8\sqrt[8]{x}},$$

$$f(x) = \frac{1}{4}\ln\frac{x^2-1}{x^2+1}, \qquad f'(x) = \frac{x}{x^4-1},$$

$$f(x) = \sqrt{x+1} - \ln(1+\sqrt{x+1}), \qquad f'(x) = \frac{1}{2(1+\sqrt{x+1})},$$

$$f(x) = \frac{1}{4}\ln\frac{2+2x}{2-2x}, \qquad f'(x) = \frac{1}{2-2x^2},$$

$$f(x) = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}, \qquad f'(x) = -\frac{1}{\cos x},$$

$$f(x) = \arcsin(\sin x - \cos x), \qquad f'(x) = \frac{\sin x + \cos x}{\sqrt{\sin 2x}},$$

$$f(x) = \operatorname{arctg}\frac{\sin x + \cos x}{\sin x - \cos x}, \qquad f'(x) = -1,$$

$$f(x) = \frac{\arcsin x}{\sqrt{1-x^2}} + \frac{1}{2}\ln\frac{1-x}{1+x}, \qquad f'(x) = \frac{x\arcsin x}{(1-x^2)^{\frac{3}{2}}},$$

$$f(x) = \ln\sqrt{\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}}, \qquad f'(x) = -\frac{1}{2x\sqrt{1-x}},$$

$$f(x) = (x-2)\sqrt{1+e^x} - \ln\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}, \qquad f'(x) = \frac{xe^x}{2\sqrt{1+e^x}},$$

$$f(x) = \frac{-\cos x}{2\sin^2 x} + \ln\sqrt{\frac{1+\cos x}{\sin x}}, \qquad f'(x) = \frac{\cos^2 x}{\sin^3 x},$$

$$f(x) = \ln\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} + 2\operatorname{arctg}\sqrt{\frac{1-x}{1+x}}, \qquad f'(x) = \frac{1}{x}\sqrt{\frac{1-x}{1+x}}.$$

Příklad 3. Určete 1000. derivaci funkcí

$$f(x) = \frac{x^2+1}{x^3-x}, \quad f^{(1000)}(x) = 1000!\left(\frac{1}{(x-1)^{1001}} + \frac{1}{(x+1)^{1001}} - \frac{1}{x^{1001}}\right)$$

$$g(x) = \frac{1}{x^2+3x+2}, \quad g^{(1000)}(x) = 1000!\left(\frac{1}{(x+1)^{1001}} - \frac{1}{(x+2)^{1001}}\right)$$

Miroslav Doupovec